Infra-red fixed point structure of soft supersymmetry breaking mass terms

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Abstract

We show that the soft SUSY breaking mass terms may have infra-red stable fixed points at which they are related to the gaugino masses and argue that in a generic unification these masses should lie close to their fixed points. We consider the implications for the family dependence of squark and slepton masses and the related flavour changing neutral currents and determine conditions under which models with flavour changing couplings and masses at a high scale may lead to a family independent effective theory at low scales. The analysis is illustrated for a variety of models for which we compute both the fixed point structure and determine the rate of approach to the fixed point.

Introduction

Our knowledge of the fundamental laws of nature are entirely consistent with the idea that what we are observing is an effective low energy theory following from some underlying stage of unification at a very high scale M_X. Provided we add a stage of low energy supersymmetry all the general features of the Standard Model including spontaneous symmetry breakdown can be understood. The light states of the theory are those that are protected from the high scale by symmetry; fermions are chiral and vectors are associated with a local gauge symmetry. The supersymmetry allows us to have light scalars provided they are associated by supersymmetry to chiral fermions. Given this very attractive picture it is natural to ask whether the low energy parameters of the theory giving the couplings masses and mixing angles can similarly be understood on the basis of the low-energy properties of the theory. The reason this may be possible is that, although the fundamental couplings should be determined by the underlying theory, their measured values correspond to their low-energy values. The renormalisation group equations which are needed to determine the low-energy values arecompletely determined in terms of the low-energy structure of the theory. Thus the initial values of the masses and couplings determined by the underlying unified(?) theory serve only to give the boundary conditions for the parameters of the effective low-energy theory. This raises the possibility that the low energy parameters may be largely determined by the dynamics of the low-energy theory itself through the infra-red fixed point structure of its renormalisation group equations, the value of a parameter (or ratio of parameters) at the fixed point being insensitive to the initial value.

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In the context of the Standard Model the only ratio of parameters that may lie close to an (approximate) infra red stable fixed point (IRSFP) is the ratio of the top Yukawa coupling to the QCD gauge coupling[1][1]. In the minimal supersymmetric extension of the model (MSSM) the ratio of both the top and bottom Yukawas may lie close to their fixed point values. In a previous paper [2] we discussed this possibility in some detail and computed how close to the fixed point the ratios are likely to be driven in going from the unification scale to the scale of electroweak breaking. We also raised the question whether the remaining couplings of the theory might be determined by the fixed points of the underlying theory lying beyond the Standard Model because we found that the rate of approach to the fixed point could be very rapid. While this does not realise the original aim of interpreting couplings as due to the dynamics at *low* scales it does raise the possibility that the couplings are determined by the dynamics of the effective theory below the Planck or string scale. The beauty of the idea is that the structure of the RG equations are determined by the multiplet structure and symmetries of the theory, features that it may be possible to determine from the structure of the theory at the electroweak scale. For example the pattern of fermion masses and mixings strongly suggests the existence of a (spontaneously broken) family symmetry above the gauge unification scale [3]. Identification of this symmetry is sufficient to determine the RG equations and thus, possibly, to completely determine the mass matrices via the IRSFP structure. In this paper we extend the discussion of the fixed point structure to include the implications for the soft SUSY breaking terms. This allows us to address the problem of flavour violation associated with any attempt to enlarge the symmetries of the Standard Model to include a family symmetry. For closely related work on the infrared structure of such soft terms see refs. [4, 5].

The general form of the effective scalar potential with soft SUSY breaking is:

$$
\begin{aligned}
V_{eff} \;=\; & \sum_i \left| \frac{\partial \tilde{W}}{\partial Z_i} \right|^2 + \sum_i |m_i|^2 |Z_i|^2 + \sum_j ((A_j \tilde{W}_{3j} + h.c.) \\
& + (B_j \tilde{W}_{2j} + h.c.) + (C_j \tilde{W}_j + h.c.)) + \text{gauge terms}
\end{aligned}
\tag{1}
$$

where $\tilde{W}$ is an effective low-energy superpotential and $\tilde{W}_{3j}$, $\tilde{W}_{2j}$ and $\tilde{W}_{1j}$ are the terms making up its trilinear, bilinear and linear parts. The parameters m_i, A_i, B_i and C_i are the soft SUSY breaking terms.

IRSFP structure.

To discuss the implications for these terms following from the IRSFP structure we start with the RG equation for the case there is a single gauge coupling corresponding to a single gauge group factor (as is the case for $SU(5)$, $SO(10)$ or as an approximation in the case of the MSSM) or a product of identical gauge groups with a permutation symmetry (e.g. $SU(3)^3$). To illustrate the general features we first consider the simple case where there is only a single generation and a single Yukawa coupling $hQ_L Q_R H$ (such as is the case if one coupling dominates). We will consider more general couplings shortly. The RG equations in this case are

$$
\frac{d\tilde{\alpha}}{dt} = -b\tilde{\alpha}^2
\tag{2}
$$

$$
\frac{dM}{dt} = -b\tilde{\alpha}M
\tag{3}
$$

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[1]Only in the absence of the $SU(2) \otimes U(1)$ factors is there a true fixed point.

$$\frac{dY}{dt} = \sum_i 2C_2(R_i)Y\tilde{\alpha} - N^Y Y^2 \tag{4}$$

$$\frac{dm_i^2}{dt} = 4C_2(R_i)\tilde{\alpha}M^2 - YN_i^m(\sum_j m_j^2 + A^2) \tag{5}$$

$$\frac{dA}{dt} = \tilde{\alpha}\sum_i 2C_2(R_i)M - YAN^Y \tag{6}$$

where $t = ln(\mu_0^2/\mu^2)$, $Y = \frac{h^2}{4\pi^2}$, $\tilde{\alpha} = \frac{g^2}{4\pi^2}$, $C_2(R_i)$ is the Casimir appropriate to the representation R_i ($C_2(R) = \frac{N^2-1}{2N}$ for R the fundamental representation of $SU(N)$) and N^Y, N_i^m counts the number of independent (wave function renormalisation) diagrams associated with the particular term.

To exhibit the fixed point structure it is convenient to form the ratios $Y/\tilde{\alpha}$, m/M and A/M. Then we easily find

$$\frac{dln(Y/\tilde{\alpha})}{dt} = (\sum_i 2C_2(R_i) + b)\tilde{\alpha} - N_I^Y Y \tag{7}$$

$$\frac{d(m_i^2/M^2)}{dt} = 4C_2(R_i)\tilde{\alpha} + 2b\tilde{\alpha}m_i^2/M^2 - YN_i^m\frac{X}{M^2} \tag{8}$$

$$\frac{d(A/M)}{dt} = \tilde{\alpha}\sum_i 2C_2(R_i) + (2b\tilde{\alpha} - YN^Y)(A/M) \tag{9}$$

where $X = (\sum_j m_j^2 + A^2)$. One may immediately see that if $(\sum_i 2C_2(R_i) + b)$ is positive there is an infra-red stable fixed point (IRSFP) in the ratio $(Y/\tilde{\alpha})$ given by

$$\left(\frac{Y}{\tilde{\alpha}}\right)^* = \frac{(\sum_i 2C_2(R_i) + b)}{N^Y} \tag{10}$$

where we adopt the convention that fixed points are denoted with a * superscript. In [2] we investigated the rate of approach to such a fixed point and concluded that it could be so rapid in many extensions of the Standard Model that the ratio of couplings would be driven very close to the fixed point even though the range over which the extension of the Standard Model applied is very small (between M_{Planck} and $M_X \approx 10^{16} GeV$). Here we note that the ratio of soft SUSY breaking mass terms may similarly be driven to IRSFP. The ratio (A/M) has an IRSFP given by

$$\left(\frac{A}{M}\right)^* = 1 \tag{11}$$

To determine the fixed point structure of the masses we assume that eqs(10,11) apply and substitute them in eqs(8,9) to rewrite the RG equations in the form

$$\frac{d(X/M^2)}{dt} = 2\tilde{\alpha}(2\sum_i C_2(R_i) - b) - ((2\sum_i C_2(R_i) + b)\frac{\sum N_i^m}{N^Y} - 2b)(X/M^2)\tilde{\alpha}$$

$$\frac{d(N_j^m m_i^2 - N_i^m m_j^2)/M^2}{dt} = 2\tilde{\alpha}(2(N_j^m C_2(R_i) - N_i^m C_2(R_j) + b(N_j^m m_i^2 - N_i^m m_j^2)/M^2) \tag{12}$$

If $((2\sum_i C_2(R_i) + b)\frac{\sum N_i^m}{N_I^Y} - 2b)$ is positive the ratio (X/M^2) has an IRSFP given by

$$\left(\frac{X}{M^2}\right)^* = \frac{4N^Y \sum_i C_2(R_i)}{\sum_i N_i^m \sum_i 2C_2(R_i) + (\sum_i N_i^m - 2bN^Y)b} \tag{13}$$

Turning to the second of eqs(12) we see it has a fixed point if $b < 0$ at

$$\frac{N_j^m m_i^2 - N_i^m m_j^2}{M^2} = \frac{2(N_j^m C_2(R_i) - N_i^m C_2(R_j))}{(-b)} \tag{14}$$

A further point of interest is the rapidity with which the couplings and masses are driven to their fixed points. In [2] we gave an analytic formula for the rate of approach of the Yukawa couplings and concluded that in many interesting models it was very rapid; hence our interest in fixed points. For the masses the analytic solution of the renormalisation group equations [6] allows us also to compute the rate of approach. The analytic result is very lengthy and will be published elsewhere; we will give the results in the specefic cases discussed later.

Flavour changing processes.

As we shall see, in many models of interest the conditions for these IRSFP are met. In them the number of independent parameters at the fixed point is dramatically reduced;in the model presented above there are just two parameters, the gauge coupling and the gaugino mass. This offers the possibility of predicting quark and lepton masses and mixing angles simply from the form of the couplings allowed by the symmetries of the theory. In order to generate mass structures it is likely that some *interactions* must be included which distinguish between generations and this raises a major difficulty in supersymmetric theories, namely the difficulty of controlling the flavour changing processes driven by the non-degeneracy of squark masses and the flavour changing A_j terms of eq(1) which arise as result of radiative corrections from these new interactions. The experimental limits on flavour changing processes puts stringent limits on the squark and slepton masses of different generations and on the A_j terms[7, 8]. The limits depend on the relative values of the soft supersymmetry breaking terms in a complicated way that is clearly discussed in [8]. For example in the case that the ratio of gaugino masses to scalar masses is small at the Planck scale the limits restrict the off diagonal squark and slepton masses at M_{GUT} (in the squark "current" basis) of the first two generations to be much less than their mean values

$$\frac{m_{\tilde{d}}^2 - m_{\tilde{s}}^2}{m_{av}^{2(d)}} \leq 8.10^{-3} \left(\frac{M_{av}^{(d)}}{1TeV}\right)^2 , \quad \frac{m_{\tilde{e}}^2 - m_{\tilde{\mu}}^2}{m_{av}^{2(l)}} \leq 10^{-1} \left(\frac{M_{av}^{(l)}}{1TeV}\right)^2 \tag{15}$$

where m_{av}, M_{av} refers to an average value taken at the low and high (GUT) scales respectively. Since these off diagonal terms are related to squark and slepton mass squared differences (in the squark mass basis) times mixing angles we see the requirement for some degree of universality in the scalar masses. Since our proposal is that IRSFP dominate through large radiative corrections we cannot rely on initial conditions to ensure this degeneracy and to avoid flavour changing neutral currents and thus it important to determine the expectation for squark mass degeneracy at the IRSFP for models of phenomenological interest.

The model presented above is very simple but it already sheds some light on this question. In particular eq(13) shows that the soft squark masses are driven to fixed points determined simply by their representation under the gauge group. Thus family independent gauge interactions actually drive the squark and slepton masses of different families to be equal at the fixed point irrespective of their initial values, reducing the generation of favour changing neutral currents at low emerges. One explanation for the observed smallness of the latter is therefore that a family independent gauge group dominates at high energies. However models capable of generating reasonable fermion masses and mixings probably need to extend the gauge group to include a family symmetry which, when broken, distinguishes between different generations. The above discussion suggests a general

condition for this to be true to the accuracy require: We consider a gauge group which has a product of factors some commuting with family indices and some family dependent. If there is some unification of these gauge factors either in the Grand Unified sense or in the superstring sense where there need not be a single gauge group one may suppose that the gauge couplings are comparable at the initial scale (the Planck scale or some Grand Unified scale above the scale at which the couplings unify). However at low scales these couplings may diverge due to different radiative corrections. If these corrections make the gauge couplings associated with the family symmetry smaller than the family independent couplings then the latter will dominate the RG evolution of the soft masses and the squark masses may indeed be driven towards degeneracy. A particularly interesting example of this is if the family gauge symmetry is Abelian. In this case the group is not asymptotically free so the coupling is driven *smaller* at low scales. Moreover the beta function for such a group is typically very large as the multiplicity of terms can be very large (for example left-handed squarks charged under the group contribute a multiplicity of 6 corresponding to three colours and two flavours) and so the Abelian family symmetry gauge coupling can be very much smaller than the family independent non- Abelian gauge coupling. We shall discuss explicit realisations of this possibility in a subsequent paper.

Having discussed the possibility that there are family dependent gauge couplings we turn now to the question that there are family dependent Yukawa couplings. To do this we must extend the model discussed above to include more than one family of quarks. Thus we consider the general Yukawa couplings of the form $h_{ijk}Q_{i,L}Q^c_{j,R}H_k$ where we have assumed $n_g \times n_g$ independent Higgs fields coupling in different terms of the mass matrix. Now the RG equations for the Yukawa couplings are simple generalisation of eq(7)

$$\frac{dln(Y_{ijk}/\tilde{\alpha})}{dt} = (\sum_i 2C_2(R_i) + b)\tilde{\alpha} - \sum_{r=i,j,k} n_r \sum_{s,t} \sqrt{Y_{rst}Y^*_{rst}}) \qquad (16)$$

Again for $\sum_i (2C_2(R_i) + b)\tilde{\alpha} > 0$ this has an IRSFP for

$$\left(\frac{Y_{ijk}}{\tilde{\alpha}}\right)^* = \frac{\sum_i 2C_2(R_i) + b}{N_T} \qquad (17)$$

where $N_T = (n_H + n_{Q_L} + n_{Q_R})$ and n_i counts the total number of wave function renormalisation graphs associated with the state i for a particular term in the RG equation. The point to note is that, since the fixed point is being driven by a family independent gauge symmetry, the Yukawa couplings are similarly driven to a family independent IRSFP. As a result, independent of the initial conditions, the theory suppresses family dependence in the Yukawa couplings too. In turn the soft SUSY breaking terms will also be driven to family independent fixed points. The A terms have the form $A_{ijk}Q_{i,L}Q^c_{j,R}H_k$. Since only wave function renormalisation enters the generalisation of eq(6) is straightforward and has a fixed point $(A_{ijk}/M)^* = 1$. Of course in a viable model all but one of the Higgs fields H_k must acquire mass at some large scale M. The remaining field to be identified with the light MSSM Higgs field will be some combination of H_k and its couplings to the quarks will generate the quark mass matrix after electroweak breaking. The essential feature of the IRSFP structure just discussed is that the rotation that diagonalises the quark mass matrix will also diagonalise the A terms since both the Yukawa couplings h_{ijk} and the A terms A_{ijk} have the same (family symmetric) form. Similarly the soft squark masses are driven to the degenerate family independent IRSFP. Thus at the IRSFP the flavour changing processes induced by the soft terms vanish. Of course if the gauge couplings are family dependent this conclusion is not true. However if, as discussed above, the family dependent coupling is driven to be negligible in the infra-red then its effects on the Yukawa couplings will also be negligible close to the fixed point leaving the family independence unchanged.

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It is perhaps worth emphasising two features of this example that ensure the suppression of flavour changing processes. Firstly it is important that the structure of Yukawa couplings allows for a family independent fixed point; if there had only been a single Higgs with large coupling to the top quarks only, then only the top squark would have had mass determined by a fixed point; the other squark's mass would have been radiatively corrected by the gauge couplings but would be determined by their initial values and would not be driven to a family independent point. The same applies to the soft A terms. In the Yukawa sector the important point is that the initial form of the Yukawa couplings has, for special values of the Yukawa couplings, an enhanced family symmetry. This symmetry is then realised at the IRSFP driven there by the radiative corrections of the family independent gauge interactions. In the case discussed here we see there is indeed a rich $SU(3)_L \otimes SU(3)_R$ family symmetry at the fixed point under which Q_L Q_R and H transform as $(\bar{3}, 1)$ $(1, 3)$ and $(3, \bar{3})$ respectively. Of course it is not necessary for the symmetry to be so large; it is easy to construct examples with just a permutation symmetry acting on the quarks. Secondly, the D-term associated with the (Abelian) family symmetry is quite dangerous for flavour changing suppression for it generates squark and slepton masses and since the gauge factor is family dependent these masses will be family dependent too. In order for the family gauge symmetry to be broken at a high scale it is necessary for the scalar fields, ϕ, $\bar{\phi}$, breaking this symmetry to acquire vevs along a D-flat direction, $<\phi> = <\bar{\phi}>$. Consider first the usual radiative breaking mechanism under which radiative corrections drive their soft masses squared, m^2, $\bar{m}^2$ negative at some scale of $O(V)$ through radiative corrections. Then it is easy to show that $<\phi> \approx <\bar{\phi}> = O(V)$ and the D-term vev is $g^2 <\phi^2 - \bar{\phi}^2> \propto (m^2 - \bar{m}^2)$. The masses given to the squarks and sleptons from the D-term are thus proportional to $(m^2 - \bar{m}^2)$ evaluated at the scale V which, if there is no cancellation or suppression, is of the order of the supersymmetry breaking scale and potentially too large. In the case a single scalar vev is induced by a Fayet Iliopoulos term [10] the situation is even worse for the D-term is proportional to m^2/g^2 with no possibility of cancellation. However the IRFP structure is such that there are good reasons why the D-tem masses may not be too large. For the first case it is quite likely that the pattern observed above will be repeated for the Yukawa couplings involving the ϕ and $\bar{\phi}$ fields and that they will be driven[2] by the gauge coupling terms in the RG equations (which are equal for conjugate representations) to equal values at an IRSFP. In this case we can even allow for *different* initial values of m and $\bar{m}$ for they too will be driven by the gauge couplings to be equal at an IRSFP. Even in the case there is only one scalar the flavour changing problem need not be severe because the ϕ (and $\bar{\phi}$) masses may be driven much smaller than the squark and slepton masses. This will happen for example in the case of the Abelian family symmetry discussed above because its gauge coupling and gaugino mass are driven very small relative to the Standard Model couplings and gaugino masses due to its large beta function. As may be seen from the discussion presented below the gaugino sets the scale for the scalar mass at the IRSFP and hence the Standard Model singlet fields will have very small scalar masses as required.

To summarise, we have demonstrated how IRSFP determine both Yukawa couplings and soft SUSY breaking terms and shown there exist a class of models which, even with the introduction of family dependent gauge couplings, may still lead to an effective low-energy lagrangian which has family independent couplings and soft SUSY breaking masses. This opens the way to construct models in which the pattern of fermion masses and mixing angles is determined by an underlying gauge symmetry yet which does not lead to large flavour violation at low energies induced by squark and slepton masses [11]. Moreover in such theories there may only be two independent parameters, the dominant gauge coupling and one gaugino mass allowing for the possible prediction of both the structure

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[2]Provided that a symmetric structure of couplings involving the ϕ and $\bar{\phi}$ fields exists

and *magnitude* of fermion masses and mixings.

We turn now to a discussion of these ideas in the context of specific models. However we will confine our attention in this paper to the expectation for the relative sizes of SUSY breaking terms following from the IRSFP structure[3] . A study in detail of the flavour violation to be expected requires the construction of a viable theory of fermion masses and this we will discuss in another paper.

IRSFP in the MSSM

In [2] we considered the IRSFP structure for the top Yukawa coupling in the MSSM. In the approximation of ignoring the $SU(2) \otimes U(1)$ couplings the theory the RG equations for the soft SUSY breaking terms are [12, 4]

$$
\begin{aligned}
\frac{dM}{dt} &= -b_3 \tilde{\alpha}_3 M \\
\frac{dB}{dt} &= -3a \tilde{\alpha}_3 A_t \\
\frac{d\tilde{A}_t}{dt} &= \tilde{\alpha}_3 \left(\frac{16}{3} - (6a - b_3) \tilde{A}_t \right) \\
\frac{d(\tilde{\mu}_2^2 - \tilde{\mu}^2)}{dt} &= \tilde{\alpha}_3 (-3a\tilde{X} + 2b_3(\tilde{\mu}_2^2 - \tilde{\mu}^2)) \\
\frac{d\tilde{m}_{t_r}^2}{dt} &= \tilde{\alpha}_3 \left(\frac{16}{3} - 2a\tilde{X} + 2b_3 \tilde{m}_{t_r}^2 \right) \\
\frac{d\tilde{m}_{Q_t}^2}{dt} &= \tilde{\alpha}_3 \left(\frac{16}{3} - a\tilde{X} + 2b_3 \tilde{m}_{Q_t}^2 \right) \\
\frac{d\tilde{\mu}_3^2}{dt} &= \tilde{\alpha}_3 (3\tilde{A}a + b_3 \tilde{\mu}_3^2) \\
\frac{d\mu^2}{dt} &= -3Y_t \mu^2
\end{aligned}
\tag{18}
$$

where $a = (\frac{Y_t}{\tilde{\alpha}_3})^*$ and the ˜ means the masses normalised to M^2, the running gluino mass, except in the case of μ_3^2 which is normalised to $M_3\mu$. Also

$$
\tilde{X} = (m_{Q_t}^2 + m_{t_R}^2 + \mu_2^2 - \mu^2 + A_t^2)/M^2
\tag{19}
$$

As discussed above to solve these equations for their IRSFP structure it is best to take linear combinations and rewrite the RG equations in the form

$$
\begin{aligned}
\frac{d\tilde{X}}{dt} &= \tilde{\alpha}_3 \left(2\frac{16}{3} - 6a\tilde{X} + 2b_3\tilde{X} + 2\frac{16}{3}\tilde{A}_t - 12a\tilde{A}_t^2 \right) \\
\frac{d(\tilde{m}_{t_r}^2 - 2\tilde{m}_{Q_t}^2)}{dt} &= \tilde{\alpha}_3 \left(-\frac{16}{3} + 2b_3(\tilde{m}_{t_R}^2 - 2\tilde{m}_{Q_t}^2) \right) \\
\frac{d(\tilde{\mu}_2^2 - \tilde{\mu}^2 - 3\tilde{m}_{Q_t}^2)}{dt} &= \tilde{\alpha}_3 \left(-3\frac{16}{3} + 2b_3(\tilde{\mu}_2^2 - \tilde{\mu}^2 - 3\tilde{m}_{Q_t}^2) \right)
\end{aligned}
\tag{20}
$$

In the case of the MSSM $b_3 = -3$ and in the neighbourhood if the IRSFP $a \approx a^* = \frac{7}{18}$ so we see that the condition for infra-red stability of *all* the soft masses is satisfied. Solving for the positions of these fixed points gives

$$
\tilde{A}_t^* = 1
$$

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[3]This will allow us to address an outstanding question, namely the implication of the infra red structure of the theory for the μ^2 term which couples the superfields H_1 and H_2 in the superpotential of the MSSM where $H_{1,2}$ contain the Higgs scalars giving the down and up quark masses respectively.

$$\tilde{X}^* = 2$$
$$(\tilde{\mu}_2^2 - \tilde{\mu}^2)^* = -\frac{7}{18}$$
$$\tilde{m}_{t_R}^{2*} \approx \frac{34}{54}$$
$$\tilde{m}_{qt}^{2*} \approx \frac{41}{54}$$
$$\tilde{\mu}_3^{2*} = \frac{7}{18}$$
$$\mu^2 = 0 \tag{21}$$

Will these fixed points be relevant to the determination of the physical parameters? To answer this we have determined the low energy parameters using the complete analytic solution of the renormalisation group equations [6]. As discussed in [2] in the limit that at the gauge coupling unification scale the ratio of the top Yukawa coupling to the gauge coupling is initially much larger than the fixed point value there is a calculable radiative correction to the fixed point value at low energy. This gives Hill's "quasi" fixed point value [9] roughly a factor of 2 larger than the true fixed point value. For the soft SUSY breaking parameters we have calculated the low energy values assuming the initial values of the top Yukawa coupling and soft terms are in the domain of attraction of the fixed point.

The rate of approach to the fixed points can be seen from the following approximate expressionsformed by expanding the analytical solution around the fixed point values.

$$\frac{A(t)}{M(t)} = 0.98 + 0.14(\frac{A}{M} - 1) - (209.55 - 70.33(\frac{A}{M} - 1))(Y_0 - Y^*)$$

$$\frac{X(t)}{M(t)^2} = 1.85 + 0.07(\frac{A}{M} - 1) + 0.01(\frac{\mu_2^2 - \mu^2}{M^2} + 0.39) + 0.10(\frac{m_q^2}{M^2} - 0.76)$$
$$+ 0.05(\frac{m_t^2}{M^2} - 0.63) + (-529.68 - 149.89(\frac{A}{M} - 1) - 39.96(\frac{\mu_2^2 - \mu^2}{M^2} + 0.39)$$
$$- 13.32(\frac{m_q^2}{M^2} - 0.76) - 26.64(\frac{m_t^2}{M^2} - 0.63))(Y_0 - Y^*)$$

$$\frac{B(t)}{M(t)} = -0.29 - 0.12(\frac{A}{M} - 1) + 0.38(\frac{B}{M} + 0.38)$$
$$- (104.77 + 35.16(\frac{A}{M} - 1))(Y_0 - Y^*)$$

$$\frac{m_q^2(t)}{M(t)^2} = 0.75 - 0.03(\frac{A}{M} - 1) + 0.10(\frac{m_q^2}{M^2} - 0.76)$$
$$+ (-27.90 + 7.52(\frac{A}{M} - 1) - 13.32(\frac{m_q^2}{M^2} - 0.76))(Y_0 - Y^*)$$

$$\frac{m_t^2(t)}{M(t)^2} = 0.51 - 0.07(\frac{A}{M} - 1) + 0.05(\frac{m_t^2}{M^2} - 0.63)$$
$$+ (-52.34 + 15.03(\frac{A}{M} - 1) - 26.64(\frac{m_t^2}{M^2} - 0.63))(Y_0 - Y^*)$$

$$\frac{(\mu_2^2 - \mu^2)(t)}{M(t)^2} = -0.39 - 0.10(\frac{A}{M} - 1) + 0.01(\frac{\mu_2^2 - \mu^2}{M^2} + 0.39)$$
$$+ (-37.81 + 22.55(\frac{A}{M} - 1) - 39.96(\frac{\mu_2^2 - \mu^2}{M^2} + 0.39))(Y_0 - Y^*) \tag{22}$$

From the first equation we see that a 100 % deviation in the initial value of A from its fixed point value results in only a 14% deviation at low energies. However the sensitivity

of A to the initial value of the Yukawa coupling is greater. Using an initial value of $\tilde{\alpha} = 1/(25\pi)$ a 100% increase of Y_0 from its fixed point corresponds to a change of - 1 in (A/M). This may be expected to be the minimum possible change if the Yukawa coupling lies at its quasi fixed point. However as the remaining equations show the sensitivity to deviations of the initial values from the fixed point values are smaller for the other quantities. This completes our discussion of the MSSM fixed point structure, some aspects of which has been discussed by other authors[4]. However as discussed above our expectation is that the IRSFP structure will be even more relevent for models beyond the Standard Model . To illustrate the expectations we consider the three models discussed in [2] based on the unification groups $SU(3)^3$, $SU(5)$ and $SO(10)$

$SU(3)^3$

The group $SU(3)^3$ has many attractions as a non-Grand-Unified extension of the Standard Model. Provided the multiplet content is chosen symmetrically, the gauge couplings above the $SU(3)^3$ unification scale, M_X, evolve together. As a result it offers an example of a theory which can be embedded in the superstring which preserves the success of the unification predictions for gauge couplings even if, as has been found to be usually the case in the compactified string theories so far analysed, the unification or compactification scale is much higher than $10^{16} GeV$. The light multiplet structure after symmetry breaking we take to be just that of the MSSM. The n_g families are contained in n_g copies of I representations where $I = ((1,3,\bar{3}) + (\bar{3},1,3) + (3,\bar{3},1))$. In addition there are two further copies of I representations which contain the Higgs fields. Taking a single dominant Yukawa coupling its fixed point is given by Table 3 for n=0 The fixed point for the A term may be immediately determined giving

$$\left(\frac{A}{M}\right)^* = 1 \tag{23}$$

Up to now we have omitted any discussion of the μ term in this model. these terms. The $SU(2)$ doublet supermultiplets containing the Higgs fields $H_{1,2}$ giving rise to down and up quarks reside in the $(1,3,\bar{3})$ representation

$$(1,3,\bar{3}) = \begin{pmatrix} H_1 H_2 L \\ E^C \nu_r N \end{pmatrix} \tag{24}$$

where L is a lepton doublet supermultiplet, E^C is the charge conjugate singlet charged lepton supermultiplet and ν_R and N are supermultiplets neutral under the Standard Model. The μ term comes from the coupling $\lambda \epsilon_{abc} \epsilon^{ijk} A_i^a B_j^b C_k^c$ where A, B and C are superfields (assumed distinct here) containing H_1, H_2 and the N scalar which acquires a vev generating the μ term, $\mu = \lambda < N >$. There are two extremes for the magnitude of this vev. It could be of $O(M_W)$ if the N field remains light. This model corresponds to the (M+1)SSM which in our opinion is disfavoured because of cosmological problems due to the domain wall production associated with the discrete symmetry present in such models[13]. Alternatively it could be of $O(M_X)$ (the maximum it could be since it breaks $SU(3)^3$). In this case λ must be very small, of $O(M_W/M_X)$. There is a very plausible origin for this[14] for if the term proportional to $\epsilon_{abc} \epsilon^{ijk} A_i^a B_j^b C_k^c / M_{Planck}$ is present in the Kahler potential. When supersymmetry is broken a term proportional to $m_{3/2} \epsilon_{abc} \epsilon^{ijk} A_i^a B_j^b C_k^c / M_{Planck}$ is generated in the superpotential corresponding to $\lambda = O(m_{3/2} M_X / M_{Planck})$. We will assume this is the mechanism that is operative in which case, due to the smallness of λ, the RG equations are little changed by loops generated by λ couplings. The things that do change are due to the induced $\mu H_1 H_2$ term. Although the N field does not acquire a VEV until the scale M_X, radiative corrections generating

9

n	$(\frac{Y_t}{\tilde{\alpha}})^*$	$(\frac{\alpha(M_X)}{\alpha(M_C)})^B$	$(\frac{\tilde{\alpha}}{Y_t})^*_{SU(3)^3}/(\frac{\tilde{\alpha}}{Y_t})^*_{MSSM}$
0	2.44	0.48	0.16
4	5.11	0.02	0.08

Table 1: $SU(3)^3$

terms with external N fields involving loop momentum above this scale are important for the external fields do not "feel" the loop momenta and may be replaced by their vevs.

Now we can discuss the RG equations for the squark mass and μ, m_1^2, m_2^2 and m_3^2 where the last three masses are the coefficients of the $\mid H_1 \mid^2$, $\mid H_2 \mid^2$ and $H_1 H_2$ terms in the scalar potential. For the purpose of illustration it is sufficient to return to our one generation example and assume that only the top Yukawa coupling is significant. The RG equations are

$$\frac{dM}{dt} = -6\tilde{\alpha}M$$

$$\frac{dm_{\tilde{q}}^2}{dt} = \frac{32}{3}\tilde{\alpha}M^2 - 3Y(2m_q^2 + \mu_2^2 + A^2 - \mu^2)$$

$$\frac{d\mu^2}{dt} = (\frac{32}{3}\tilde{\alpha} - 3Y)\mu^2$$

$$\frac{d\mu_2^2}{dt} = \frac{32}{3}\tilde{\alpha}(M^2 + \mu^2) - 3Y(\mu_2^2 + 2m_q^2 + A^2)$$

$$\frac{d\mu_1^2}{dt} = \frac{32}{3}\tilde{\alpha}(M^2 + \mu^2) - 3Y\mu^2$$

$$\frac{d\mu_3^2}{dt} = (\frac{16}{3}\tilde{\alpha} - \frac{3}{2}Y)\mu_3^2 - \frac{32}{3}M\mu\tilde{\alpha} + 3\mu AY \qquad (25)$$

This has IRSFP

$$\left(\frac{M}{\mu}\right)^* = 0$$

$$\left(\frac{2m_{\tilde{q}}^2 + \mu_2^2 - \mu^2}{\mu^2}\right)^* = 0$$

$$\left(\frac{\mu_{1,2}^2}{\mu^2}\right)^* = 1$$

$$\left(\frac{\mu_3^2}{\mu^2}\right)^* = 0, \quad \left(\frac{\mu M}{\mu_3^2}\right)^* = 0 \qquad (26)$$

Thus an hierarchy of masses develops with μ, $\mu_{1,2}$ and μ_3 growing roughly at the same rate while $m_{\tilde{q}}^2$ and $\mu_2^2 - \mu^2$ tend to constant values and M and A fall towards zero. In this case only A is in the domain of attraction of the Standard Model fixed point (the $SU(3)^3$ fixed point is actually at it). However the combination of masses $X = 2m_{\tilde{q}}^2 + \mu_2^2 - \mu^2$ does also lie in the domain of attraction of the MSSM fixed point and should also closely approach it.

The rate of approach to this fixed points can be seen from the following expressions taken from the analytical solution to the renormalization group equations. Expanding the solution around the fixed point values we get:

$$\frac{A(t)}{M(t)} = 1 + 0.62(\frac{A}{M} - 1) - (40.13 + 32.09(\frac{A}{M} - 1))(Y_0 - Y^*)$$

n	$(\frac{Y_t}{\tilde{\alpha}})^*$	$(\frac{\alpha(M_X)}{\alpha(M_C)})^B$	$(\frac{\tilde{\alpha}}{Y_t})^*_{SU(5)}/(\frac{\tilde{\alpha}}{Y_t})^*_{MSSM}$
0	1.80	0.62	0.22
4	3.57	0.29	0.11

Table 2: SU(5)

$$
\frac{X(t)}{\mu^2(t)} = 0.47\frac{m_{\tilde{t}}^2}{\mu^2} + 0.47\frac{m_q^2}{\mu^2} + 0.47\frac{\mu_2^2}{\mu^2} + 0.18\frac{AM}{\mu^2} + 0.59\frac{M^2}{\mu^2}
$$
$$
- (16.17\frac{m_{\tilde{t}}^2}{\mu^2} + 16.17\frac{m_q^2}{\mu^2} + 16.17\frac{\mu_2^2}{\mu^2} + 18.93\frac{AM}{\mu^2} + 5.44\frac{M^2}{\mu^2})(Y_0 - Y^*)
$$
$$
\frac{m_q^2(t)}{\mu^2(t)} = 0.47\frac{m_q^2}{\mu^2} - 0.04\frac{AM}{\mu^2} + 0.17\frac{M^2}{\mu^2} - (16.17\frac{m_q^2}{\mu^2} + 0.78\frac{AM}{\mu^2} - 1.05\frac{M^2}{\mu^2})(Y_0 - Y^*)
$$
$$
\frac{\mu_2^2(t)}{\mu^2(t)} = 1 + 0.47\frac{\mu_2^2}{\mu^2} - 0.04\frac{AM}{\mu^2} + 0.17\frac{M^2}{\mu^2} - (16.17\frac{m_q^2}{\mu^2} + 0.78\frac{AM}{\mu^2} - 1.05\frac{M^2}{\mu^2})(Y_0 - Y^*)
$$
$$(27)$$

The rate of approach to the fixed point is not very great. However if we add to our theory n copies of chiral superfields in $(I + \bar{I})$ representations things change dramatically as was discussed in [2]. The factor determining the closeness of approach to the fixed point of the Yukawa couplings is given by the third column of Table 3 and for n=4 the approach is within 2% compared to 48% for n=0. The reasons were discussed in [2] but largely follow from the fact that the additional matter makes the couplings run fast. For the case of $n = 4$ the rate of approach to the fixed point is given by the following expressions:

$$
\frac{A(t)}{M(t)} = 1 + 0.39(\frac{A}{M} - 1) - (5.40 + 3.97(\frac{A}{M} - 1))(Y_0 - Y^*)
$$
$$
\frac{X(t)}{\mu^2(t)} = 0.09\frac{m_{\tilde{t}}^2}{\mu^2} + 0.09\frac{m_q^2}{\mu^2} + 0.09\frac{\mu_2^2}{\mu^2} + 0.04\frac{AM}{\mu^2} + 0.11\frac{M^2}{\mu^2}
$$
$$
- (0.61\frac{m_{\tilde{t}}^2}{\mu^2} + 0.61\frac{m_q^2}{\mu^2} + 0.61\frac{\mu_2^2}{\mu^2} - 0.18\frac{AM}{\mu^2} + 0.03\frac{M^2}{\mu^2})(Y_0 - Y^*)
$$
$$
\frac{m_q^2(t)}{\mu^2(t)} = 0.09\frac{m_q^2}{\mu^2} - 0.02\frac{AM}{\mu^2} + 0.03\frac{M^2}{\mu^2} - (0.61\frac{m_q^2}{\mu^2} - 0.12\frac{AM}{\mu^2} + 0.003\frac{M^2}{\mu^2})(Y_0 - Y^*)
$$
$$
\frac{\mu_2^2(t)}{\mu^2(t)} = 1 + 0.09\frac{\mu_2^2}{\mu^2} - 0.02\frac{AM}{\mu^2} + 0.03\frac{M^2}{\mu^2} - (0.61\frac{m_q^2}{\mu^2} - 0.12\frac{AM}{\mu^2} + 0.003\frac{M^2}{\mu^2})(Y_0 - Y^*)
$$
$$(28)$$

It can be seen that the effect of additional matter dramatically speeds up the rate of approach to the fixed points of the soft terms too.

SU(5)

In our next example we will consider an SU(5) model in which matter is arranged in three generations in $I = \{\psi^{xy}(10) + \phi_x(\bar{5})\}$ representations together with n further copies of chiral superfields in $(I + \bar{I})$ representations plus a Higgs sector made up of a (complex) adjoint, $\Sigma(24)$, to break SU(5) and a set of Weinberg-Salam 5-plets $H_1(5) + H_2(\bar{5})$. We consider the case of a single large Yukawa coupling leading to the top quark mass. The Yukawa coupling IRFP structure and the rate of approach is summarised in Table 3 [2].

11

n	$(\frac{Y_t}{\tilde{\alpha}})^*$	$(\frac{\alpha(M_X)}{\alpha(M_C)})^B$	$(\frac{\tilde{\alpha}}{Y_t})^*_{SO(10)}/(\frac{\tilde{\alpha}}{Y_t})^*_{MSSM}$
0	1.75	0.51	0.22
4	2.89	0.24	0.13

Table 3: SO(10)

For brevity we only give here the expansion about the fixed point for the soft terms for the case n=4 :

$$
\begin{aligned}
\frac{A(t)}{M(t)} &= 1 + 0.37(\frac{A}{M} - 1) - (33.66 + 21.82(\frac{A}{M} - 1))(Y_0 - Y^*) \\
\frac{X(t)}{\mu^2(t)} &= 0.19\frac{m_t^2}{\mu^2} + 0.19\frac{m_q^2}{\mu^2} + 0.31\frac{\mu_2^2}{\mu^2} + 0.04\frac{AM}{\mu^2} + 0.56\frac{M^2}{\mu^2} \\
&\quad - (11.24\frac{m_t^2}{\mu^2} + 11.24\frac{m_q^2}{\mu^2} + 6.24\frac{\mu_2^2}{\mu^2} + 9.69\frac{AM}{\mu^2} + 2.86\frac{M^2}{\mu^2})(Y_0 - Y^*) \\
\frac{m_q^2(t)}{\mu^2(t)} &= 0.18\frac{m_q^2}{\mu^2} - 0.06\frac{AM}{\mu^2} + 0.19\frac{M^2}{\mu^2} - (11.24\frac{m_q^2}{\mu^2} - 1.23\frac{AM}{\mu^2} + 0.01\frac{M^2}{\mu^2})(Y_0 - Y^*) \\
\frac{\mu_2^2(t)}{\mu^2(t)} &= 1 + 0.31\frac{\mu_2^2}{\mu^2} - 0.05\frac{AM}{\mu^2} + 0.08\frac{M^2}{\mu^2} - (6.24\frac{m_q^2}{\mu^2} - 0.98\frac{AM}{\mu^2} + 1.42\frac{M^2}{\mu^2})(Y_0 - Y^*)
\end{aligned}
\tag{29}
$$

As in the $SU(3)^3$ case the effect of additional matter is to speed up the approach to the fixed point for the soft terms too..

SO(10)

Finally for $SO(10)$ with three families and n=4 vectorlike $(16 + \bar{16})$ representation the IRSFP structure and the rate of approach to it is summarised in Table 3 and the following equations. In this example we have included the additional 16, 45 and 10 dimensional vectorlike representations needed to break the group.

$$
\begin{aligned}
\frac{A(t)}{M(t)} &= 1 + 0.37(\frac{A}{M} - 1) - (33.66 + 21.82(\frac{A}{M} - 1))(Y_0 - Y^*) \\
\frac{X(t)}{\mu^2(t)} &= 0.19\frac{m_t^2}{\mu^2} + 0.19\frac{m_q^2}{\mu^2} + 0.31\frac{\mu_2^2}{\mu^2} + 0.04\frac{AM}{\mu^2} + 0.76\frac{M^2}{\mu^2} \\
&\quad - (11.24\frac{m_t^2}{\mu^2} + 11.24\frac{m_q^2}{\mu^2} + 6.24\frac{\mu_2^2}{\mu^2} + 9.69\frac{AM}{\mu^2} + 6.29\frac{M^2}{\mu^2})(Y_0 - Y^*) \\
\frac{m_q^2(t)}{\mu^2(t)} &= 0.18\frac{m_q^2}{\mu^2} - 0.06\frac{AM}{\mu^2} + 0.21\frac{M^2}{\mu^2} - (11.24\frac{m_q^2}{\mu^2} - 1.23\frac{AM}{\mu^2} + 0.89\frac{M^2}{\mu^2})(Y_0 - Y^*) \\
\frac{\mu_2^2(t)}{\mu^2(t)} &= 1 + 0.31\frac{\mu_2^2}{\mu^2} - 0.05\frac{AM}{\mu^2} + 0.16\frac{M^2}{\mu^2} - (6.24\frac{m_q^2}{\mu^2} - 0.98\frac{AM}{\mu^2} + 0.71\frac{M^2}{\mu^2})(Y_0 - Y^*)
\end{aligned}
\tag{30}
$$

Again one may see the relatively rapid approach to the fixed points.

To summarise we have considered the implications for the soft SUSY breaking terms of the IRSFP structure in a variety of models. We have argued that such structure is likely to be relevant for a wide variety of models beyond the Standard Model with couplings and masses driven very close to the fixed point. A study of the family dependent effects

12

shows that in a sub-class of models the resultant flavour changing neutral processes are very small at the fixed points even in theories with family dependent interactions offering the possibility of constructing viable models of family interactions capable of generating the observed fermion mass structure. We have determined the fixed point predictions for the soft SUSY breaking terms for a variety of models and also the rate of approach to the fixed point structure. This shows that some subset of the soft terms will be determined quite well by the fixed point structure substantially limiting the parameter space of the low-energy effective supersymmetric theory.

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